EC 831: Empirical Methods in Macroeconomics

Aeimit Lakdawala

Michigan State University

State Space Model with Regime Switching

Consider the model

$$
\begin{aligned}
y_t &= H_{s_t}\beta_t + A_{s_t}z_t + e_t \\
\beta_t &= F_{s_t}\beta_{t-1} + v_t \\
\begin{pmatrix} e_t \\ v_t \end{pmatrix} &\sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} R_{s_t} & 0 \\ 0 & Q_{s_t} \end{pmatrix}\right)
\end{aligned}
$$

where

$$p(s_t = j | s_{t-1} = i) = p_{ij} \quad i, j = 1, 2, \ldots M$$

Bayesian Estimation: Gibbs Sampler

$$\theta = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$$

$$\begin{aligned}
\theta_1 &= [R_i, Q_i] \quad i = 1, 2, ...M \\
\theta_2 &= [A_i, H_i, F_i] \quad i = 1, 2, ...M \\
\theta_3 &= [p_{ij}] \quad i = 1, 2, ...M, j = 1, 2, ...M \\
\theta_4 &= (s_1, s_2, \ldots, s_T) \\
\theta_5 &= (\beta_1, \beta_2, \ldots \beta_T)
\end{aligned}$$

Gibbs Sampler

1) Draw θ_1 from $p(\theta_1|\theta_2, \theta_3, \theta_4, \theta_5, y, z)$

Prior: R_i and Q_i (for each i) $\sim$ Inverse Wishart

Conditional on $\theta_2, \theta_3, \theta_4, \theta_5, y, z$ this is similar to estimating variance of two (independent) system of linear equations with normal errors

Posterior will be Inverse Wishart (see VAR notes for details)

- Note the dependence on s_t

Gibbs Sampler

2) Draw θ_2 from $p(\theta_2|\theta_1, \theta_3, \theta_4, \theta_5, y, z)$

Prior: A_i, F_i and H_i (for each i) $\sim$ Normal

Conditional on $\theta_1, \theta_3, \theta_4, \theta_5, y, z$ this is similar to estimating coefficients of two (independent) system of linear equations with normal errors

Posterior will be Normal (see VAR notes for details)

- Note again the dependence on s_t

Gibbs Sampler

3) Draw θ_3 from $p(\theta_3|\theta_1, \theta_2, \theta_4, \theta_5, y, z)$

Prior: $p_{ij}\ \forall i, j \sim$ Dirichlet

Conditional on $\theta_1, \theta_2, \theta_4, \theta_5, y, z$ this involves estimating the transition matrix parameters

Posterior will be Dirichlet (see regime-switching notes for details)

Gibbs Sampler

4) Draw θ_4 from $p(\theta_4|\theta_1, \theta_2, \theta_3, \theta_5, y, z)$

Conditional on $\theta_1, \theta_2, \theta_3, \theta_5, y, z$ this is involves estimating the indicator variable

See the regime class notes for details

$$p(s^T|\psi_T) = p(s_T|\psi_T) \prod_{t=1}^{T} p(s_t|s_{t+1}, \psi_t)$$

Gibbs Sampler

5) Draw θ_5 from $p(\theta_5|\theta_1, \theta_2, \theta_3, \theta_4, y, z)$

Conditional on $\theta_1, \theta_2, \theta_3, \theta_4, y, z$ this involves estimating the continuous latent variable

See the linear state space notes for details on Carter-Kohn simulation smoother