EC 831: Empirical Methods in Macroeconomics

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Primiceri (2005)

Time Varying Structural Vector Autoregressions and Monetary Policy

following the notation in the paper

$$y_t = c_t + B_{1,t} y_{t-1} + B_{2,t} y_{t-2} + u_t$$

$$Var(u_t) = \Omega_t$$

Stacking the coefficients into B_t and the lags into

$$X_t' = I_n \otimes [1, y_{t-1}', y_{t-2}']$$

we get

$$y_t = X_t' B_t + u_t$$

Triangular decomposition of variance matrix Ω_t

$$\Omega_t = A_t^{-1}\Sigma_t\Sigma_t' A_t^{-1'}$$

$$A_t = \begin{pmatrix} 1 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{pmatrix}$$

$$\Sigma_t = \begin{pmatrix} \sigma_{1,t} & 0 & 0 \\ 0 & \sigma_{2,t} & 0 \\ 0 & 0 & \sigma_{3,t} \end{pmatrix}$$

$$y_t = X_t' B_t + A_t^{-1} \Sigma_t \varepsilon_t \qquad \varepsilon_t \sim N(0,1)$$

$$\begin{aligned} B_t &= B_{t-1} + \nu_t \\ \alpha_t &= \alpha_{t-1} + \zeta_t \\ log(\sigma_t) &= log(\sigma_{t-1}) + \eta_t \end{aligned}$$

$$Var \begin{pmatrix} \varepsilon_t \\ \nu_t \\ \zeta_t \\ \eta_t \end{pmatrix} = \left[\begin{pmatrix} I_n & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{pmatrix} \right]$$

Priors

$$
\begin{aligned}
B_0 &\sim N(\widehat{B}_{OLS}, 4 \cdot V(\widehat{B}_{OLS})),\\
A_0 &\sim N(\widehat{A}_{OLS}, 4 \cdot V(\widehat{A}_{OLS})),\\
\log \sigma_0 &\sim N(\log \widehat{\sigma}_{OLS}, I_n),\\
Q &\sim IW(k_Q^2 \cdot 40 \cdot V(\widehat{B}_{OLS}), 40),\\
W &\sim IW(k_W^2 \cdot 4 \cdot I_n, 4),\\
S_1 &\sim IW(k_S^2 \cdot 2 \cdot V(\widehat{A}_{1,OLS}), 2),\\
S_2 &\sim IW(k_S^2 \cdot 3 \cdot V(\widehat{A}_{2,OLS}), 3),
\end{aligned}
$$

Consider

$$k_Q = \{0.01; 0.05; 0.1\}$$

$$k_S = \{0.01; 0.1; 1\}$$

$$k_W = \{0.001; 0.01\}$$

Pick $k_Q = 0.01$, $k_S = 0.1$ and $k_W = 0.01$

- Model Selection technique known as Reversible Jump Markov Chain Monte Carlo (RJMCMC)

Standard Deviation of Residuals

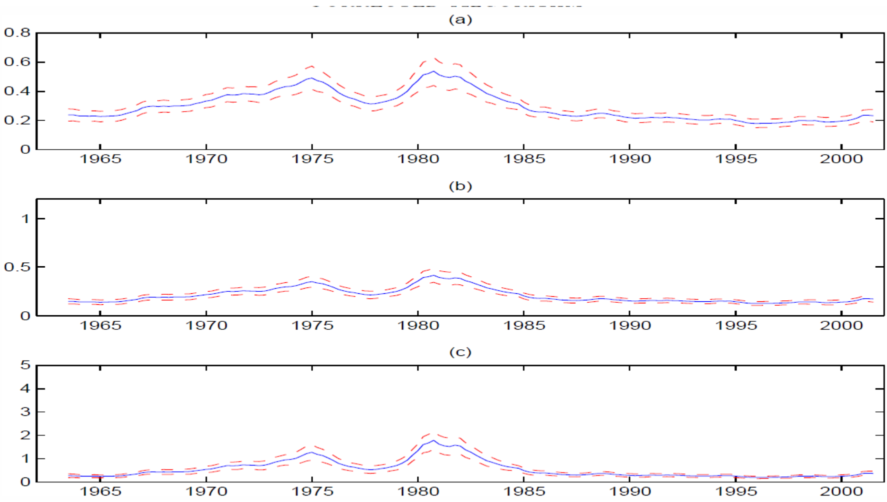


Figure: a)Inflation, b)Unemployment, c)Interest Rate

Impulse Response of Inflation: Monetary Policy Shock

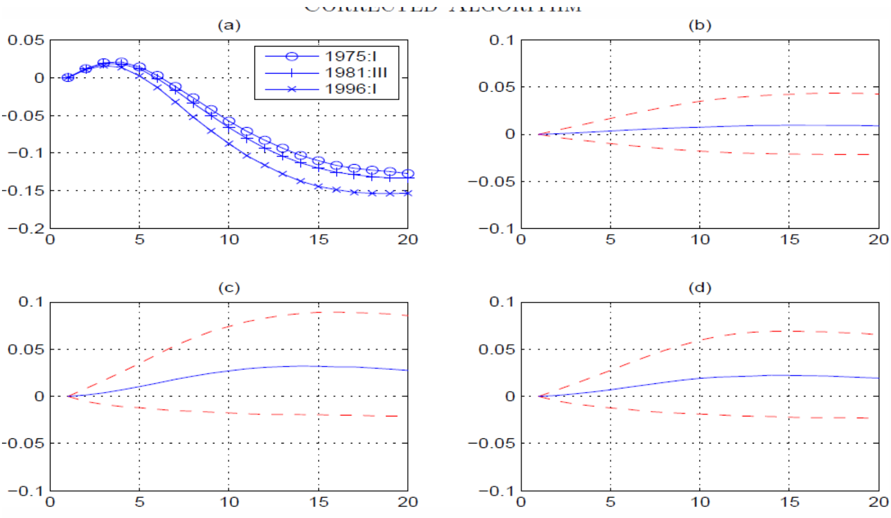


Figure: (b) difference between 1975:I and 1981:III, (c)1975:I and 1996:I, (d) 1981:III and 1996:I

Impulse Response of Unemployment: Monetary Policy Shock

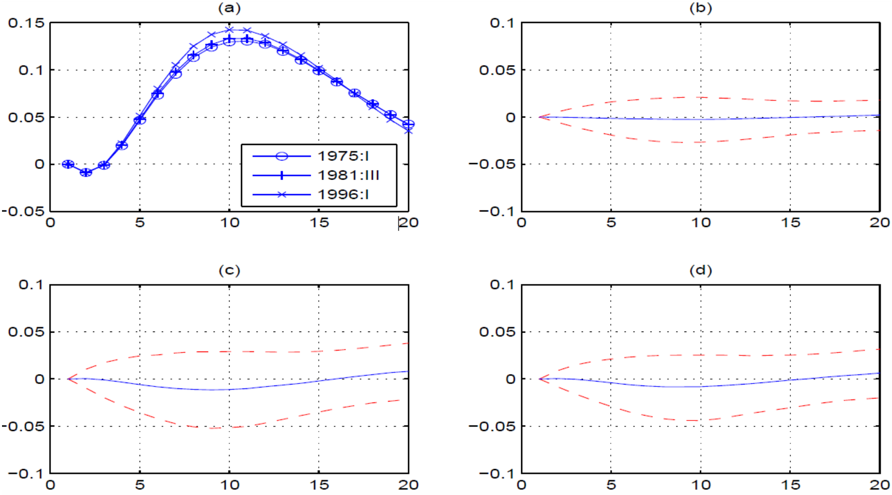
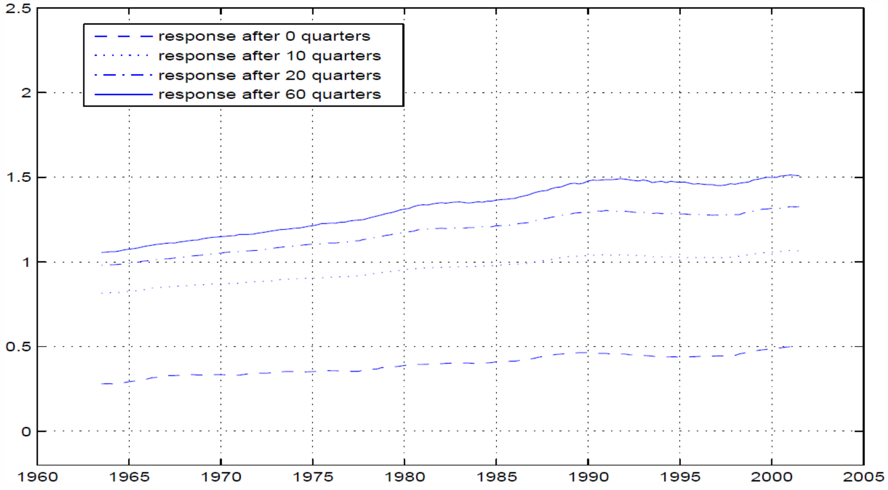


Figure: (b) difference between 1975:I and 1981:III, (c)1975:I and 1996:I, (d) 1981:III and 1996:I

Interest rate response to 1% increase in inflation



Interest rate response to 1% increase in inflation

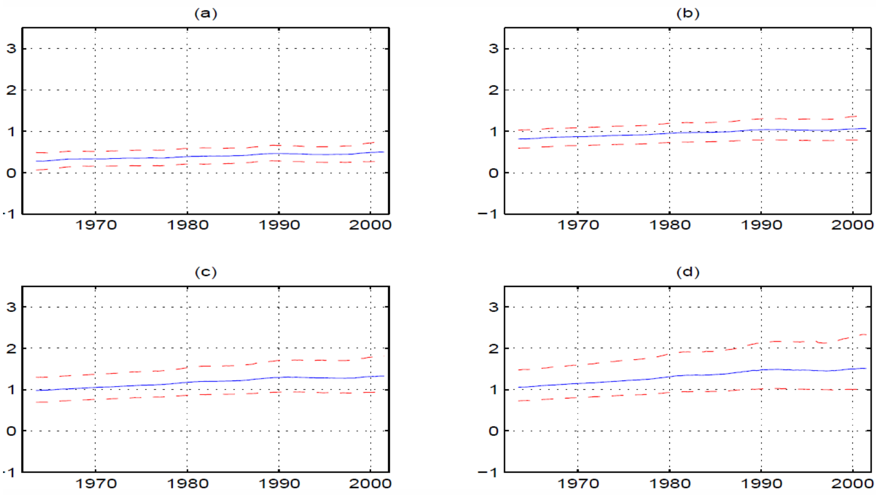
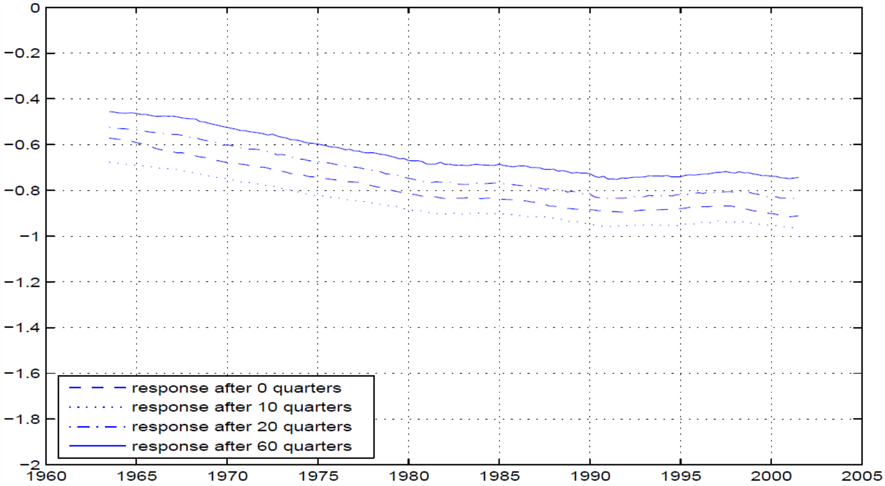


Figure: (a) Simultaneous response, (b) response after 10 quarters, (c) response after 20 quarters, (d) response after 60 quarters.

Interest rate response to 1% increase in unemployment



Interest rate response to 1% increase in unemployment

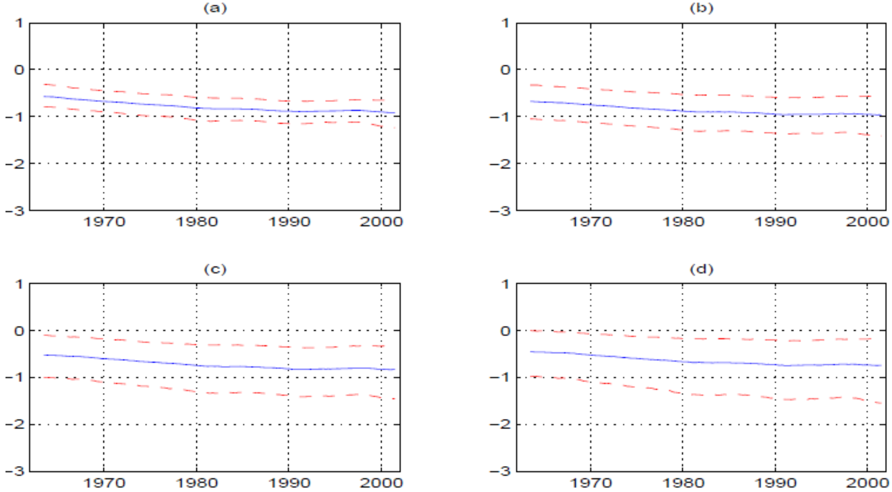


Figure: (a) Simultaneous response, (b) response after 10 quarters, (c) response after 20 quarters, (d) response after 60 quarters.

Counterfactual: Monetary policy rule of ’91-’92 in ’70s

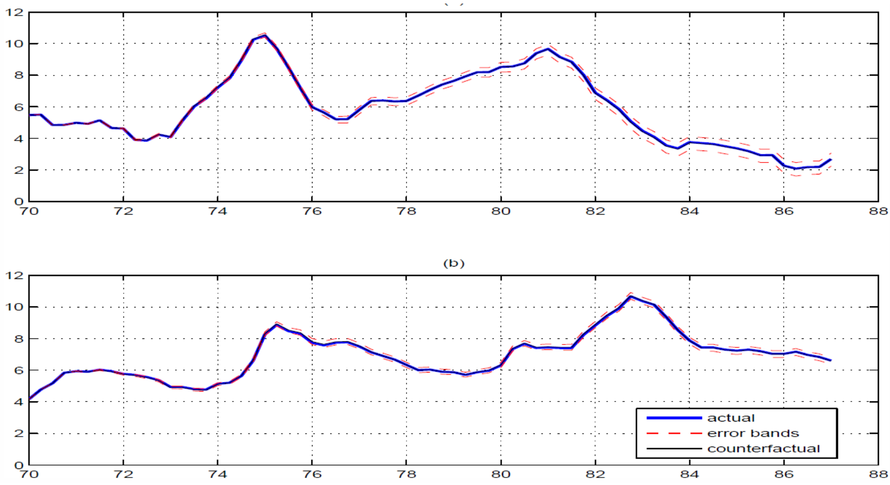


Figure: a)Inflation, b)Unemployment

"...variation in the variance covariance matrix crucial for analyzing the dynamics of the contemporaneous relations..."

"...little evidence for a causal link between changes in interest rate systematic responses and the high inflation and unemployment episodes"

Original Algorithm in Primiceri (2005) paper

The parameters are

- Σ^T
- B^T
- A^T
- V
- s^T

Let $\theta = (B^T, A^T, V)$.

So now we have 3 blocks

1. θ
2. s^T
3. Σ^T

Original Algorithm in Primiceri (2005) paper

1. Draw Σ^T from $p(\Sigma^T|y^T, \theta, s^T)$
2. Draw s^T from $p(s^T|y^T, \Sigma^T, \theta)$
3. Draw θ from $p(\theta|y^T, \Sigma^T)$

What is wrong here?

Original Algorithm in Primiceri (2005) paper

1. Draw Σ^T from $p(\Sigma^T|y^T,\theta,s^T)$
2. Draw s^T from $p(s^T|y^T,\Sigma^T,\theta)$
3. Draw θ from $p(\theta|y^T,\Sigma^T)$

What is wrong here?

The last step should be 3) Draw θ from $p(\theta|y^T,\Sigma^T,s^T)$

Compare with univariate stochastic volatility algorithm

Can we just replace the last step with 3) Draw θ from $p(\theta|y^T,\Sigma^T,s^T)$?

Original Algorithm in Primiceri (2005) paper

1. Draw Σ^T from $p(\Sigma^T|y^T, \theta, s^T)$
2. Draw s^T from $p(s^T|y^T, \Sigma^T, \theta)$
3. Draw θ from $p(\theta|y^T, \Sigma^T)$

What is wrong here?

The last step should be 3) Draw θ from $p(\theta|y^T, \Sigma^T, s^T)$

Compare with univariate stochastic volatility algorithm

Can we just replace the last step with 3) Draw θ from $p(\theta|y^T, \Sigma^T, s^T)$?

No. Let's see why.

Original Algorithm in Primiceri (2005) paper

The model is

$$y_t = X_t' B_t + A_t^{-1} \Sigma_t \varepsilon_t \qquad \varepsilon_t \sim N(0,1)$$

We transform in the following way

$$A_t(y_t - X_t' B_t) = \widetilde{y}_t = \Sigma_t \varepsilon_t$$

Now square both sides and take logs

$$\begin{aligned} log(\widetilde{y}_t^2) &= 2log(\sigma_t) + log(\varepsilon_t^2) \\ y_t^* &= 2log(\sigma_t) + \varepsilon_t^* \end{aligned}$$

$$\begin{aligned} \varepsilon_t^* | s_t = i &\sim N(m_i - 1.2704, v_i^2) \\ P(s_t = i) &= q_i \end{aligned}$$

Original Algorithm in Primiceri (2005) paper

So conditional on s_t, ε_t^* is normal

$$\varepsilon_t^*|s_t = i \sim N(m_i - 1.2704, v_i^2)$$

But conditional on s_t, ε_t is not normal

$$\varepsilon_t|s_t = \sqrt{\exp\left(\varepsilon_t^*\right)}|s_t \sim \quad ?$$

Corrected Algorithm

Just switch the order

1. Draw Σ^T from $p(\Sigma^T | y^T, \theta, s^T)$
2. Draw θ from $p(\theta | y^T, \Sigma^T)$
3. Draw s^T from $p(s^T | y^T, \Sigma^T, \theta)$

where the old algorithm was

1. Draw Σ^T from $p(\Sigma^T | y^T, \theta, s^T)$
2. Draw s^T from $p(s^T | y^T, \Sigma^T, \theta)$
3. Draw θ from $p(\theta | y^T, \Sigma^T)$

Corrected Algorithm

Why does this work? The trick is to use a different blocking scheme:

Consider the blocks 1)Σ^T and 2) (s^T, θ). We want draws from

1. $p(\Sigma^T | y^T, \{\theta, s^T\})$ and
2. $p(\{\theta, s^T\} | y^T, \Sigma^T)$

Now factor the joint posterior $p(\{\theta, s^T\} | y^T, \Sigma^T)$ into

- $p(\theta | y^T, \Sigma^T) p(s^T | \theta, y^T, \Sigma^T)$

So we can draw from the marginal and conditional to get a draw from the joint

Corrected Algorithm

1. Draw Σ^T from $p(\Sigma^T | y^T, \theta, s^T)$
2. Draw θ from $p(\theta | y^T, \Sigma^T)$
3. Draw s^T from $p(s^T | y^T, \Sigma^T, \theta)$

The key is to draw Σ^T right after drawing s^T.